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The Eccentric-Distance Sum of Cycles and Related Graphs

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Abstract

Let $G = (V, E)$ be a simple connected graph. The eccentric-distance sum of G is defined as $\xi^{ds}(G) = \sum_{u \in V(G)} e(u)D(u)$ where $e(u)$ is the eccentricity of the vertex u in G and $D(u)$ is the sum of distances between u and all other vertices of G. In this paper, we establish formulae to calculate the eccentric-distance sum for some cycle related graphs, namely C_n, complement of C_n, shadow of C_n and the line graph of C_n. Also, it is shown that, the eccentric-distance sum of C_n is less than the eccentric-distance sum of shadow of C_n for all $n \geq 3$.

Key words: Distance, Eccentricity, Eccentric-Distance Sum.

AMS Classification 2010: 05C12.

1. Introduction

By a graph $G = (V, E)$, we mean a finite undirected connected graph without loops or multiple edges. The *order* and *size* of G are denoted by n and p respectively. For basic definitions and terminologies we refer to [1]. For vertices u and v in a connected graph G, the distance $d(u, v)$ is the length of a shortest $u - v$ path in G. A $u - v$ path of length $d(u, v)$ is called a $u - v$ *geodesic*. The *eccentricity* $e(v)$ of a vertex v in G is the maximum distance from v and a vertex of G. The minimum eccentricity among the vertices of G is the *radius*, *rad* G or $r(G)$ and the maximum eccentricity is its *diameter*, *diam* G of G. A $u - v$ walk of G is a finite, alternating sequence $u = u_0 e_1 u_1 e_2 \cdots , e_n u_n = v$ of vertices and edges in G beginning with vertex u and ending with vertex v such that $e_i = u_{i-1} u_i$, $i = 1, 2, \cdots , n$. The number n is called the *length* of the walk. A walk in which all the vertices are distinct is called a *path*. A closed walk $u_0, u_1, u_2, \cdots , u_n$ in which $n \geq 3$ and $u_0, u_1, u_2, \cdots , u_{n-1}$ are distinct is called a *cycle* of length n and is denoted by C_n. The complement $\bar{G}$ of a simple graph G is a simple graph with vertex set V, two vertices being adjacent in $\bar{G}$ if and only if they are not adjacent in G. The line graph $L(G)$ is a graph in which

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the vertices are the lines of G and two points in $L(G)$ are adjacent if and only if the corresponding lines are adjacent in G. The *shadow graph* $S(G)$ of a connected graph G is constructed by taking two copies of G say $G^{'}$ and $G^{''}$. Join each vertex $u^{'}$ in $G^{'}$ to the neighbours of the corresponding vertex $u^{''}$ in $G^{''}$. The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G(V, E)$ where $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$. The sum $G_1 + G_2$ is the graph $G_1 \cup G_2$ together with all the lines joining points of V_1 to the points of V_2 . In [2], Gupta, Singh and Madan introduced a novel topological descriptor which is called eccentric-distance sum index (EDS) and then the concept was studied by various authors. The eccentric-distance sum of G is defined as $\xi^{ds}(G) = \sum\limits_{u \in V(G)} e(u)D(u)$ where $e(u)$ is the eccentricity of the vertex u in G and $D(u)$ is the sum of distances between u and all other vertices of G. In this paper, we establish formulae to calculate the eccentric-distance sum for some cycle related graphs, namely C_n, complement of C_n, Shadow of C_n and the line graph of C_n. Throughout this paper G denotes a connected graph with at least three vertices. $\xi^{ds}(K_n) = n(n-1)$. $L(G)$ is isomorphic to G if and only if G is a cycle.

2. Main results

Theorem 2.1 The eccentric distance sum of, the sum of two cycles of length n is $\xi^{ds}(C_n + C_n) = 2n \times \lfloor n/2 \rfloor \times [n + (\sum\limits_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor)]$

Proof: Clearly the graph $C_n + C_n$ has $2n$ number of vertices.

$$e(v_i) = \lfloor n/2 \rfloor \text{ where } i = 1, 2, 3, \ldots, 2n$$

$$\begin{aligned} D(v_i) &= 1 + 1 + 2 + \cdots + \lfloor (n-1)/2 \rfloor + \lfloor n/2 \rfloor + \underbrace{(1 + 1 + \cdots + 1)}_{(n \text{ times})} \\ &= 0 + 0 + 1 + 1 + 2 + \cdots + \lfloor (n-1)/2 \rfloor + \lfloor n/2 \rfloor + n(1) \\ &= [0 + 0 + 1 + 1 + 2 + \cdots + \lfloor (n-1)/2 \rfloor + \lfloor n/2 \rfloor] + n \\ &= [\sum_{j=1}^{n+1} \lfloor (j-1)/2 \rfloor] + n \end{aligned}$$

$$\begin{aligned} \xi^{ds}(C_n + C_n) &= \sum_{i=1}^{2n} e(v_i)D(v_i) \\ &= e(v_1)D(v_1) + \cdots + e(v_{2n})D(v_{2n}) \end{aligned}$$

$$= \lfloor n/2 \rfloor [(\sum_{j=1}^{n+1} \lfloor (j-1)/2 \rfloor) + n] + \cdots + \lfloor n/2 \rfloor [(\sum_{j=1}^{n+1} \lfloor (j-1)/2 \rfloor) + n] (2n \text{ times})$$

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$$= 2n\lfloor n/2\rfloor[(\sum_{j=1}^{n+1}\lfloor(j-1)/2)\rfloor)+n]$$

Hence $\xi^{ds}(C_n+C_n) = 2n\times\lfloor n/2\rfloor\times[n+(\sum_{i=1}^{n+1}\lfloor(i-1)/2)\rfloor)]$.

Remark 2.2 $\xi^{ds}(C_n) = n\times\lfloor n/2\rfloor\times(\sum_{i=1}^{n+1}\lfloor(i-1)/2\rfloor)$ Proof: The eccentricity of any vertex in (C_n+C_n) is same as the eccentricity of any vertex in C_n. Also, the distance sum of any vertex in (C_n+C_n) is equal to n plus the distance sum of any vertex in C_n. Thus $\xi^{ds}(C_n) = n\times\lfloor n/2\rfloor\times(\sum_{i=1}^{n+1}\lfloor(i-1)/2\rfloor)$.

Theorem 2.3 The eccentric distance sum of the sum of two cycles of length n and m where $n\neq m$ is $\xi^{ds}(C_n+C_m) = n\times\lfloor n/2\rfloor\times[m+(\sum_{i=1}^{n+1}\lfloor(i-1)/2\rfloor)]+m\times\lfloor m/2\rfloor\times[n+(\sum_{i=1}^{m+1}\lfloor(i-1)/2\rfloor)]$ Proof: Consider the graph C_n+C_m where $n\neq m$
Clearly it contains $n+m$ number of vertices.

$$e(v_i) = \lfloor n/2\rfloor \text{for all } i = 1,2,3,\ldots,n$$

$$e(v_i) = \lfloor m/2\rfloor \text{ for all } i = n+1,\ldots,m$$

$$D(v_i) = 1+1+2+\cdots+\lfloor(n-1)/2\rfloor+\lfloor n/2\rfloor+\underbrace{(1+1+\cdots+1)}_{(m\text{ times})} \quad \text{for all } i = 1,2,3,\cdots,n$$

$$= 0+0+1+1+2+\cdots+\lfloor(n-1)/2\rfloor+\lfloor n/2\rfloor+\underbrace{(1+1+\cdots+1)}_{(m\text{ times})}$$

$$= [\sum_{j=1}^{n+1}\lfloor(j-1)/2\rfloor]+m \quad \text{for all } i = 1,2,3,\ldots,n$$

$$D(v_i) = 1+1+2+\cdots+\lfloor(m-1)/2\rfloor+\lfloor m/2\rfloor+\underbrace{(1+1+\cdots+1)}_{(n\text{ times})} \quad \text{for all } i = n+1,\ldots,m$$

$$= 0+0+1+1+2+\cdots+\lfloor(m-1)/2\rfloor+\lfloor m/2\rfloor+\underbrace{(1+1+\cdots+1)}_{(n\text{ times})}$$

$$= [\sum_{j=1}^{m+1}\lfloor(j-1)/2\rfloor]+n \; for\ all\ i = n+1,\ldots,m$$

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$$\xi^{ds}(C_n + C_m) = \sum_{i=1}^{n+m} e(v_i)D(v_i)$$
$$= e(v_1)D(v_1) + \cdots + e(v_n)D(v_n) + e(v_{n+1})D(v_{n+1}) + \cdots + e(v_m)D(v_m)$$
$$= \lfloor n/2 \rfloor [(\sum_{j=1}^{n+1} \lfloor (j-1)/2 \rfloor) + m] + \cdots + \lfloor n/2 \rfloor [(\sum_{j=1}^{n+1} \lfloor (j-1)/2 \rfloor) + m] +$$
$$\lfloor m/2 \rfloor [(\sum_{j=1}^{m+1} \lfloor (j-1)/2 \rfloor) + n] + \cdots + \lfloor m/2 \rfloor [(\sum_{j=1}^{m+1} \lfloor (j-1)/2 \rfloor) + n]$$
$$= n \times \lfloor n/2 \rfloor \times [m + \sum_{j=1}^{n+1} \lfloor (j-1)/2 \rfloor] + m \times \lfloor m/2 \rfloor \times [n + (\sum_{i=1}^{m+1} \lfloor (i-1)/2 \rfloor)]$$

Hence
$\xi^{ds}(C_n + C_m) = n \times \lfloor n/2 \rfloor [m + (\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor)] + m \times \lfloor m/2 \rfloor [n + (\sum_{i=1}^{m+1} \lfloor (i-1)/2 \rfloor)]$.

Remark 2.4 $\xi^{ds}(C_n + C_m) \neq \xi^{ds}(C_{n+m})$. Proof: By remark 2.2,$\xi^{ds}(C_{n+m}) = (n + m) \times \lfloor (n+m)/2 \rfloor \times (\sum_{i=1}^{n+m+1} \lfloor (i-1)/2 \rfloor)$
By theorem 2.3,$\xi^{ds}(C_n + C_m) = n \times \lfloor n/2 \rfloor \times [m + (\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor)] + m \times \lfloor m/2 \rfloor \times [n + (\sum_{i=1}^{m+1} \lfloor (i-1)/2 \rfloor)]$
Hence the result follows.

Theorem 2.5 For $n \geq 5, \xi^{ds}(\overline{C_n}) = 2n(n+1)$. Proof: $e(v_i) = 2$ for all $i = 1, 2, \cdots, n$

$$D(v_i) = n + 1 \text{ for all } i = 1, 2, \cdots, n$$
$$\xi^{ds}(\overline{C_n}) = \sum_{i=1}^{n} e(v_i)D(v_i)$$
$$= e(v_1)D(v_1) + \cdots + e(v_n)D(v_n)$$
$$= 2(n+1) + \cdots + 2(n+1)(n \text{ times})$$
$$= n \times 2 \times (n+1) = 2n(n+1).$$

Remark 2.6 For $n = 3, 4, (\overline{C_n})$ is a disconnected graph and so eccentric distance sum cannot be determined.

Remark 2.7 Eccentric distance sum cannot be determined for $(\overline{C_n + C_n})$. Proof: $(\overline{C_n + C_n})$ is the union of $(\overline{C_n})$ and $(\overline{C_n})$.

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That is $(\overline{C_n + C_n}) = (\overline{C_n}) \cup (\overline{C_n})$
$(\overline{C_n}) \cup (\overline{C_n})$ is a disconnected graph.
Thus the result follows.

Theorem 2.8 For $n \geq 6$, $\xi^{ds}(\overline{C_n}) < \xi^{ds}(C_n)$. Proof: For $n \geq 6$, $n+1 < \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor$

$$\Rightarrow \; n(n+1) < n \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor$$

$$\Rightarrow 2n(n+1) < 2n \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor$$

$$< n \lfloor n/2 \rfloor \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor$$

$$\Rightarrow 2n(n+1) < n \lfloor n/2 \rfloor \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor$$

Thus $\xi^{ds}(\overline{C_n}) < \xi^{ds}(C_n)$ for $n \geq 6$.

Theorem 2.9 If two graphs are isomorphic then their eccentric distance sum is equal. Proof: Let $G_1$and G_2be two graphs which are isomorphic. Then the eccentricity of every vertex in $G_1$and G_2 will be equal and the distance sum of every vertex in $G_1$and G_2will be equal. Hence the eccentric distance sum of the two graphs will be equal.

Result 2.10 $\xi^{ds}(C_n + C_n) = \xi^{ds}(K_{2n})$ for $n = 3$. Proof: The graph $C_3 + C_3$ is isomorphic to the complete graph with six vertices
K_6.Thus $\xi^{ds}(C_3 + C_3) = \xi^{ds}(K_6)$.

We can prove the same result by giving particular value for $n = 3$
We know that $\xi^{ds}(C_n + C_n) = 2n \times \lfloor n/2 \rfloor \times [n + (\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor)]$

$\xi^{ds}(C_3 + C_3) = 2 \times 3 \times \lfloor 3/2 \rfloor [3 + 0 + 0 + 1 + 1] = 30$

We know that $\xi^{ds}(K_n) = n(n-1)$

$$\xi^{ds}(K_6) = 6(6-1) = 30$$
$$\xi^{ds}(C_3 + C_3) = \xi^{ds}(K_6).$$

Result 2.11 For $n = 5$,$\xi^{ds}(C_n) = \xi^{ds}(\overline{C_n})$. Proof: The cycle graph on 5 vertices, C_5 is the unique self- complementary graph (up to graph isomorphism)

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That is C_5 is isomorphic to its complement.
Thus $\xi^{ds}(C_5) = \xi^{ds}(\overline{C_5})$ Also, We can show the same result by giving particular value for $n = 5$ in the formula

$$\xi^{ds}(C_n) = n \times \lfloor n/2 \rfloor \times [\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor]$$

$$\xi^{ds}(C_5) = 5 \times \lfloor 5/2 \rfloor \times [\sum_{i=1}^{6} \lfloor (i-1)/2 \rfloor]$$

$$= 5 \times 2 \times [0 + 0 + 1 + 1 + 2 + 2] = 60$$

$$\xi^{ds}(\overline{C_n}) = 2n(n+1) = 60$$

$$\xi^{ds}(C_5) = \xi^{ds}(\overline{C_5}).$$

Theorem 2.12 $\xi^{ds}(C_n) = \xi^{ds}(L(C_n))$. Proof: By observation 1.2, C_n is isomorphic to $L(C_n)$. Thus $\xi^{ds}(C_n) = \xi^{ds}(L(C_n))$.

Theorem 2.13 For $n = 3$, $\xi^{ds}(S(C_n)) = 4n \lceil n/2 \rceil [1 + \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor]$. Proof:
$e(v_i) = \lceil n/2 \rceil$ for all $i = 1, 2, 3, \cdots, n$

$$e(v_i^{'}) = \lceil n/2 \rceil \text{ for all } i = 1, 2, \cdots, n$$

$$D(v_i) = [2 \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor] + 2 \text{ for all } i = 1, 2, 3, \cdots, n$$

$$= 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1]$$

$$D(v_i^{'}) = [2 \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor] + 2 \text{ for all } i = 1, 2, 3, \cdots, n$$

$$= 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1]$$

For $n = 3$

$$\xi^{ds}(S(C_n)) = \sum_{u \in V(S(C_n))} e(u)D(u)$$

$$= e(v_1)D(v_1) + \cdots + e(v_n)D(v_n) + e(v_1^{'})D(v_1^{'}) + \cdots + e(v_n^{'})D(v_n^{'})$$

$$= \lceil n/2 \rceil \times 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1] + \cdots + \lceil n/2 \rceil \times 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) +1] +$$

$$\lceil n/2 \rceil \times 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1] + \cdots + \lceil n/2 \rceil \times 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1]$$

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$$= 2n[\lceil n/2\rceil \times 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2\rfloor) + 1]]$$

$$= 4n[\lceil n/2\rceil \, [(\sum_{i=1}^{n+1} \lfloor (i-1)/2\rfloor) + 1]]$$

Hence $\xi^{ds}(S(C_n)) = 4n \lceil n/2\rceil \, [1 + \sum_{i=1}^{n+1} \lfloor (i-1)/2\rfloor]$.

Theorem 2.14 For $n \geq 4$, $\xi^{ds}(S(C_n)) = 4n \times \lfloor n/2\rfloor \times [1 + (\sum_{i=1}^{n+1} \lfloor (i-1)/2\rfloor)]$. Proof: Clearly $S(C_n)$ has $2n$ number of vertices

$e(v_i) = \lfloor n/2\rfloor$ for all $i = 1, 2, 3, \cdots, n$

$e(v_i^{'}) = \lfloor n/2\rfloor$ for all $i = 1, 2, \cdots, n$

$$D(v_i) = [2(\sum_{i=1}^{n+1} \lfloor (i-1)/2\rfloor)] + 2 \text{ for all } i = 1, 2, 3, \cdots, n$$

$$= 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2\rfloor) + 1]$$

$$D(v_i^{'}) = [2(\sum_{i=1}^{n+1} \lfloor (i-1)/2\rfloor)] + 2 \text{ for all } i = 1, 2, 3, \cdots, n$$

$$= 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2\rfloor) + 1]$$

$$\xi^{ds}(S(C_n)) = \sum_{u \in V(S(C_n))} e(u)D(u)$$

$$= e(v_1)D(v_1) + \cdots + e(v_n)D(v_n) + e(v_1^{'})D(v_1^{'}) + \cdots + e(v_n^{'})D(v_n^{'})$$

$$= \lfloor n/2\rfloor \times 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2\rfloor) + 1] + \cdots + \lfloor n/2\rfloor \times 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2\rfloor) + 1] + \lfloor n/2\rfloor \times 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2\rfloor) + 1] + \cdots + \lfloor n/2\rfloor \times 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2\rfloor) + 1]$$

$$= 2n[\lfloor n/2\rfloor \times 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2\rfloor) + 1]]$$

$$= 4n \lfloor n/2\rfloor \, [(\sum_{i=1}^{n+1} \lfloor (i-1)/2\rfloor) + 1]$$

Hence $\xi^{ds}(S(C_n)) = 4n \times \lfloor n/2\rfloor \times [1 + (\sum_{i=1}^{n+1} \lfloor (i-1)/2\rfloor)]$.

Theorem 2.15 $\xi^{ds}(C_n) < \xi^{ds}(S(C_n))$ for $n \geq 3$. Proof: First we prove for $n \geq 4$.

$\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor < \ 1 + \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor$

$\lfloor n/2 \rfloor \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor < \ \lfloor n/2 \rfloor [1 + \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor]$

$n \lfloor n/2 \rfloor \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor < \ n \lfloor n/2 \rfloor [1+ \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor] < 4n \lfloor n/2 \rfloor [1+ \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor]$

i.e. $n \lfloor n/2 \rfloor \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor < \ 4n \lfloor n/2 \rfloor [1 + \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor]$

$\Rightarrow \xi^{ds}(C_n) < \ \xi^{ds}(S(C_n)) forn \geq 4$ For $n = 3$

$$\begin{aligned}
\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor &< 1 + \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor \\
\lfloor n/2 \rfloor \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor &< \ \lfloor n/2 \rfloor [1 + \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor] \\
&\leq \ \lceil n/2 \rceil [1 + \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor] \quad (\text{since} \quad \lfloor n/2 \rfloor \leq \lceil n/2 \rceil) \\
\lfloor n/2 \rfloor \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor &< \ \lceil n/2 \rceil [1 + \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor] \\
n \lfloor n/2 \rfloor \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor &< n \lceil n/2 \rceil [1 + \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor] \\
&< 4n \lceil n/2 \rceil [1 + \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor] \\
n \lfloor n/2 \rfloor \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor &< 4n \lceil n/2 \rceil [1 + \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor]
\end{aligned}$$

Thus $\xi^{ds}(C_n) < \xi^{ds}(S(C_n))$ for $n \geq 3$.

Theorem 2.16 $\xi^{ds}(S(\overline{C_n})) = 8n(n+2)$ for $n \geq 5$. Proof: $S(\overline{C_n})$ has $2n$ vertices

$$\begin{aligned}
e(v_i) &= 2 \quad \text{for all} \quad i = 1, 2, 3, \cdots, n \\
e(v_i^{'}) &= 2 \quad \text{for all} \quad i = 1, 2, \cdots, n \\
D(v_i) &= 2(n+2) \quad \text{for all} \quad i = 1, 2, 3, \cdots, n \\
D(v_i^{'}) &= 2(n+2) \quad \text{for all} \quad i = 1, 2, 3, \cdots, n
\end{aligned}$$

$$\begin{aligned}\xi^{ds}(S(\overline{C_n})) = &\sum_{u\in V(S(\overline{C_n}))} e(u)D(u)\\ = &\ e(v_1)D(v_1)+\cdots+e(v_n)D(v_n)+e(v_1^{'})D(v_1^{'})+\cdots+e(v_n^{'})D(v_n^{'})\\ = &\ 2[2(n+2)]+\cdots+2[2(n+2)]+2[2(n+2)]+\cdots+2[2(n+2)]\\ = &\ 2n[2\times(2(n+2))] \qquad = 8n(n+2).\end{aligned}$$

Result 2.17 $\xi^{ds}(S(\overline{C_n})) = \xi^{ds}(S(C_n))$ for $n = 5$. Proof: Since $C_n$is isomorphic to its complement , the result follows.

$$\begin{aligned}\xi^{ds}(S(\overline{C_n})) &= 8n(n+2)\\ \xi^{ds}(S(\overline{C_5})) &= 8\times 5(5+2)\\ &= 280 \qquad (1)\\ \xi^{ds}(S(C_n)) &= 4n\lfloor n/2\rfloor[(\sum_{i=1}^{n+1}\lfloor (i-1)/2\rfloor)+1]\\ \xi^{ds}(S(C_5)) &= 4\times 5\times\lfloor 5/2\rfloor[(\sum_{i=1}^{6}\lfloor (i-1)/2\rfloor)+1]\\ &= 4\times 5\times 2[0+0+1+1+2+2+1]\\ &= 280 \qquad (2)\end{aligned}$$

From (1) and (2)
$\xi^{ds}(S(\overline{C_n})) = \xi^{ds}(S(C_n))$ for $n = 5$.

Result 2.18 $\xi^{ds}(S(\overline{C_n})) < \xi^{ds}(S(C_n))$ for $n \geq 6$ Proof: We find the values of $\xi^{ds}(S(\overline{C_n}))$ and$\xi^{ds}(S(C_n))$ as follows:
When $n = 6$, $\xi^{ds}(S(\overline{C_n})) = 8n(n+2) = 384$

$$\xi^{ds}(S(C_n)) = 4n\lfloor n/2\rfloor[(\sum_{i=1}^{n+1}\lfloor (i-1)/2\rfloor)+1] = 720$$

When $n = 7$, $\xi^{ds}(S(\overline{C_n})) = 8n(n+2) = 504$

$$\xi^{ds}(S(C_n)) = 4n\lfloor n/2\rfloor[(\sum_{i=1}^{n+1}\lfloor (i-1)/2\rfloor)+1] = 1092$$

When $n = 8$, $\xi^{ds}(S(\overline{C_n})) = 8n(n+2) = 640$

$$\xi^{ds}(S(C_n)) = 4n\lfloor n/2\rfloor[(\sum_{i=1}^{n+1}\lfloor (i-1)/2\rfloor)+1] = 2176$$

When $n = 9$, $\xi^{ds}(S(\overline{C_n})) = 8n(n+2) = 792$

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$$\xi^{ds}(S(C_n)) = 4n\lfloor n/2 \rfloor [(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1] = 3024$$

Thus we see that $\xi^{ds}(S(\overline{C_n})) < \xi^{ds}(S(C_n))$ for $n \geq 6$.

3. Conclusion

In this paper we have found the eccentric distance sum of, the sum of two cycles of length n, the eccentric distance sum of a cycle , the eccentric distance sum of complement of a cycle , the eccentric distance sum of the line graph of a cycle , the eccentric distance sum of the shadow graph of a cycle and we conclude that the eccentric distance sum of the complement of a cycle is less than the eccentric distance sum of a cycle for $n \geq 6$, the eccentric distance sum of a cycle is less than the eccentric distance sum of the shadow of a cycle for $n \geq 3$ and the eccentric distance sum of the shadow of complement of a cycle is less than the eccentric distance sum of the shadow of a cycle for $n \geq 6$.

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